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(6) Endogenous Growth Theory:

Arrow, Romer and Lucas

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Recall that in the Harrod-Domar, Kaldor-Robinson, Solow-Swan and the Cass-Koopmans growth models, we have maintained, either explicitly or implicitly, that technical change is "exogenous". In the Schumpeter version, this was not true: we had "swarms" of inventors arising under particular conditions. The Smithian and Ricardian models also had technical change arising from profit-squeezes or, in the particular case of Smith, arising because of previous technical conditions.

Allyn A. Young (1928) had argued for the resurrection of the Smithian concept in terms of increasing returns to scale: division of labor induces growth which enables further division of labor and thus even faster growth. The idea that technological change is induced by previous economic conditions one may term "endogenous growth theory".

The need for a theory of technical change was there: according to some rather famous calculations from Solow (1957), 87.5% of growth in output in the United States between the years 1909 and 1949 could be ascribed to technological improvements alone. Hence, what is called the "Solow Residual" - the g(A) term in the growth equation given earlier, is enormous. One of the first reactions was to argue that by reducing much of that influence to pure capital improvements, capital-intensity seem to play a larger role than imagined in these 1957 calculations - Solow does go on to argue, for instance, that increased capital-intensive investment embodies new machinery and new ideas as well as increased learning for even further economic progress (Solow, 1960).

However, Nicholas Kaldor was really the first post-war theorist to consider endogenous technical change. In a series of papers, including a famous 1962 one with J.A. Mirrlees, Kaldor posited the existence of a "technical progress" function. that per capita income was indeed an increasing function of per capita investment. Thus "learning" was regarded as a function of the rate of increase in investment. However, Kaldor held that productivity increases had a concave nature (i.e. increases in labor productivity diminish as the rate of investment increases). This proposition, of course, falls short of Solow's insistence on constant returns. asdsadasdasda

K.J. Arrow (1962) took on the view that the level of the "learning" coefficient is a function of cumulative investment (i.e. past gross investment). Unlike Kaldor, Arrow sought to associate the learning function not with the rate of growth in investment but rather with the absolute level of knowledge already accumulated. Because Arrow claimed that new machines are improved and more productive versions of those in existence, investment does not only induce productivity growth of labor on existing capital (as Kaldor would have it), but it would also improve the productivity of labor upon all subsequent machines made in the economy.

The trick is to utilize the concept that while firms face constant returns, the industry or economy as a whole takes increasing returns to account. This can be easily formalized. Taking our old Cobb-Douglas production function, Y = AKaL1-a, there is constant returns to scale for all inputs together (since a + (1-a) = 1). Therefore, as noted in the Solow model, it might seem as if output per capital and consumption per capita does not grow unless the exogenous factor, A, grows too. To endogenize A, let us first establish the Cobb-Douglas production function for each individual firm:

Yi = AiKia Li1-a

where, one can note, the output of an individual firm is related with capital, labor as well as the "augmentation" of labor by Ai. Arrow (1962) assumed that Ai, the technical augmentation factor, might thus written look specific to the firm, but it is in fact related to total "knowledge" in the economy. This knowledge and experience, Arrow argued, is common to all firms: a free and public good (i.e. non-competitive consumption).

So the first question is how knowledge is accumulated. Arrow argued that it arises from past cumulative investment of all firms. Let us call this cumulative investment G. Thus, Arrow assumed that the technical augmentation factor is related to economy-wide aggregate capital in a process of "learning-by-doing". In other words, the experience of the particular firm is related to the stock of total capital in the economy, G, by the function:

Ai = Gz

thus, as the physical capital stock G accumulates, knowledge used by a particular firm also accumulates by a proportion z such that 0 Yi = GzKia Li1-a

where, note, only G does not have a subscript i (i.e. is not particular to the firm), it is a productive force external to the firms (i.e. a Marshallian externality) and assumed a free public good. This force is free and any firm employing it will not implicate on another firm's consumption: it is freely-available knowledge. Thus, we still maintain CRS at the firm level, but in the aggregate, however, G = K - since it is only the accumulated stock of capital for the economy. Therefore, the "economy-wide" aggregate production function is:

Y = Ka + zL1-a

Arrow (1962) assumed that a + z <1. Therefore, increasing only capital (or only labor) does not lead to increasing returns. We can obtain increasing returns to scale as a + z + (1-a)="z"> 0, but capital and labor must both expand. However, by adding this restriction, Arrow's original model exhibits non-increasing returns to scale in aggregate if the rate of growth in an economy is steady.

Paul Romer (1986) went to great lengths to disqualify the restriction imposed by Arrow. Taking the Arrow idea of disembodied knowledge, Romer concluded that there indeed could be constant returns, but Romer claims that the rate of growth of K alone may yield increasing returns, i.e. he assumed a + z > 1 was possible. With this externality, the growth rate of capital gK is equal to the growth rate of per capita consumption gc = gC - gL such that: gK = gc > 0 i.e. there is constant positive growth in per capita consumption and capital. This is obvious when we examine our general Ramsey-Cass-Koopmans consumption growth equation: gc = e-1[? k - n - d] What is ? k? Well, recall that k = K/L, so: Yi = KiaLi1-aGt becomes: yi = kiaGt so: ? ik = akia-1Gt But as, in aggregate, Gt = Kt = (kL)t, this can be rewritten as: ? ik = akia-1(kL)t = akia+t-1Lt where the marginal product that rules is that of the firm and so is with respect to ka only (as opposed to ka+t, as the firm cannot "capture" the externality). So, plugging in, our steady-state growth rate of consumption is: gc = e-1[akia+t-1Lt - n - d] It is the externality, from learning-by-doing, that gives a positive growth rate for consumption and output. So even if the rate of capital growth is merely sufficient to cover capital-widening investment (i.e. gK = gL = n + d), the indirect effect K has upon Y through G is more than sufficient to ensure an increase in per capita income, capital and consumption just as Solow originally imagined it perhaps should - people are constantly made better off. In addition, we should note that the higher the level of disembodied knowledge, the more "soil" exists upon which innovation (i.e. increases in productivity) can work and the higher the rate of technical progress. Hence, if growth is to be fostered by technological improvement (i.e. greater productivity), such is enhanced, by implication, in capital-intensive production processes. There is an unfortunate element to the Romer equation - namely, the fact that the size of the labor force enters as an argument. Thus, if L increases, then the growth rate of consumption rises. This is a rather unattractive feature which could be solved if we had originally divided our accumulated knowledge equation by L, so that: would be used instead. However, the notation in the model gets unecessarily complicated after this. Finally, note that the social planner's solution to the Arrow-Romer model would be different from the competitive solution just presented. Namely, a social planner's marginal product of capital would be different as he would take the externality into account. Thus, under a social planner, we would obtain the following consumption growth equation: gc = e-1[(a+t)kia+t-1Lt - n - d] which would yield a higher steady-state growth rate. Thus, the equlibrium growth rate under a competitive system is smaller than the optimal growth rate under a social planner. This is obvious as the firms are not, individually, taking the externality into account.

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